

[Time:2.30 Hrs]

[Marks:75]

Please check whether you have got the right question paper.

- N.B:
1. All question are compulsory.
 2. Figures to the right indicate full marks.
 3. Students answering in the regional language should refer in case of doubt to the main text of the paper in English.

Q.1 Attempt any THREE of the following:

- A) Show that the following system of equations is consistent and solve it
 $2x + 5y = 1, 3x + 2y = 7.$ 5
- B) Find the inverse of the matrix by adjoint method: 5
- $$A = \begin{bmatrix} 3 & -10 & -1 \\ -2 & 8 & 2 \\ 2 & -4 & -2 \end{bmatrix}$$
- C) Find the rank of the matrix $A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & -1 & 3 \\ -2 & 8 & 2 \end{bmatrix}.$ 5
- D) Express $\sin 3\theta$ and $\cos 3\theta$ in terms of powers of $\cos \theta$ and $\sin \theta$. 5
- E) Prove that $\cosh^{-1}x = \log(x + \sqrt{x^2 - 1})$. 5
- F) Prove that $\text{Log}(1 + i) = \log \sqrt{2} + i(2n\pi + \frac{\pi}{4})$. 5

Q.2 Attempt any THREE of the following:

- A) Solve $(y^2 + 2xy)dx + x^2dy = 0$. 5
- B) Solve $2xydx + (x^2 + 3y^2)dy = 0$. 5
- C) Solve $(x+9y-7)dx = (2x+3y-6)dy$. 5
- D) Solve $xyp^3 + (x^2 - 2y^2)p - 2pxy = 0$. 5
- E) Solve $(D^2 + 4D + 3)y = e^{-2x}$ 5
- F) Solve $\frac{dy}{dx} - y \tan x = -y^2 \sec x$. 5

Q.3 Attempt any THREE of the following:

- A) Find the Laplace transform of $\sin^3 2t$. 5
- B) Find the Laplace transform of $t^2 \sin at$. 5
- C) Verify the convolution theorem for the pair of functions:
 $f_1(t) = t, f_2(t) = e^{at}.$ 5

- D) Evaluate $\int_0^{\infty} e^t \left(\frac{\cos at - \cos bt}{t} \right) dt.$ 5
- E) Find the inverse transforms of $\frac{S+7}{S^2+2S+5}.$ 5
- F) Find the inverse Laplace transform of $\tan^{-1}\left(\frac{2}{S^2}\right).$ 5

Q.4 Attempt any THREE of the following:

- A) Evaluate $\int (x^2 - y^2) dA$ over the area of the triangle whose vertices are the points (0,1), (1,1) and (1,2). 5
- B) Change the order of integrals and evaluate $\int_0^{\infty} \int_0^{\infty} e^{-xy} \sin nx \, dx dy.$ 5
- C) Evaluate $\int_0^{\pi} \int_0^{2\pi} \int_0^1 r^4 \sin \theta \, dr \, d\theta \, d\phi$ 5
- D) Compute $\int \int \int \frac{dx \, dy \, dz}{(x+y+z+1)^3}$ if the region of integration is bounded by the coordinate planes and the plane $x + y + z = 1.$ 5
- E) Find the area enclosed by the pair of curves $y = 2 - x$ and $y^2 = 2(2 - x)$ using double integration. 5
- F) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0.$ 5

Q.5 Attempt any THREE of the following:

- A) Evaluate $\int_0^{\infty} \frac{x^4}{4^x} dx$ 5
- B) Evaluate $\int_0^{\infty} x^7 e^{-2x^2} dx.$ 5
- C) Evaluate $\int_0^1 x^3 (1 - \sqrt{x})^5 dx.$ 5
- D) Compute $\int_0^{\pi/2} \sqrt{\tan \theta} \, d\theta.$ 5
- E) Evaluate $\int_0^{\infty} \frac{e^{-x}}{x} (1 - e^{-ax}) dx,$ Where $(a > -1).$ 5
- F) Verify the rule of differentiation under integral sign for the integral $\int_0^{a^2} \log(ax) \, dx.$ 5
